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Appendix† to 'Boundary-layer separation at a free streamline. Part 2.'

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When $\eta \rightarrow \infty$, three linearly independent complementary solutions of (3·4) and (3·5) have asymptotic expansions starting with multiples of

$$\eta^{\frac{2}{3}}, \quad \eta^{\frac{1}{3}(4\alpha+5)}, \quad \eta^{-\frac{1}{3}(4\alpha+8)} \exp\{\tfrac{15}{32} A_0 \eta^{\frac{8}{3}}\}, \tag{A1}$$

where $\alpha = n\gamma$ or 1 corresponding to $F_{n\gamma}$ or F_1, respectively, and $A_0 = 1{\cdot}9507\ldots$ (see I). The boundary condition (3·7) requires that the exponentially large term in (A1) be absent from the solutions sought. We may verify that the largest term of any particular solution of (3·4) or (3·5), when $\eta \rightarrow \infty$, will also be the exponential in (A1) since (3·7) is satisfied by F_0 and F_γ.

This term may be eliminated from both complementary and particular solutions by integrating backwards, starting at $n_0 = 10$ (say), using any reasonable values for the function and its first derivative, and choosing the second derivative to satisfy the differential equation with the third derivative omitted. The basis for this approximation is the WKBJ method which indicates that the exponentially large term results from the retention of the highest derivative. This procedure is not exact and small errors are introduced into the values of the second derivative initially. However, the errors decay quickly and for values of $\eta < 6$, which are the ones required here, the results are quite adequate.

The following numerical procedure was followed. We note that F_0' is a complementary solution of (3.4) and (3.5). Denoting another complementary and particular solution by ${}_cF_\alpha$ and ${}_pF_\alpha$, respectively, we integrate backwards from η_0 to $\eta = 0$. The solution we seek must be expressible in the form

$$F_\alpha(\eta) = {}_pF_\alpha(\eta) + AF_0'(\eta) + B_cF_\alpha(\eta), \tag{A2}$$

where A and B are constants of integration. Noting $F_0'(0) = 0$ and applying the boundary conditions (3·6) we find

$$B = -{}_pF_\alpha(0)/{}_cF_\alpha(0), \tag{A3}$$

and

$$A = -[{}_pF_\alpha'(0) + B_cF_\alpha'(0)]/F_0''(0). \tag{A4}$$

These values for A and B are now used to compute initial values at η_0 for a new particular solution using (A2). A second particular solution is computed by integrating backwards and we re-solve for A and B. After two iterations the particular solution satisfies (3·6) to $O(10^{-6})$. For $\eta < 6$, the numerical values for $F_\alpha(\eta)$ and its first two derivatives were accurate to at least 4 significant figures.

† This Appendix was referred to immediately after equations (3·8) in Ackerberg (1971) and should have appeared at the end of that paper.

In determining the eigenfunction $F_\gamma(\eta)$, ${}_pF_\gamma(\eta) \equiv 0$ and the complementary solution will satisfy

$$ {}_cF_\gamma(0) = 0, $$

if the correct eigenvalue, determined in I, is used in the differential equation. The constants A and B are related using the condition $F'_\gamma(0) = 0$, i.e.

$$ A = -B_c F'_\gamma(0)/F''_0(0). \tag{A5} $$

A single multiplicative constant remains, and we choose it so that

$$ F''_\gamma(0) \equiv 1. $$

Caution is required in selecting a method for the numerical integrations. A one-step technique, such as the Runge–Kutta method, does not work well because of the very small step size required to eliminate large truncation errors due to the exponential in (A1) with large argument of $O(1000)$ when $\eta \approx \eta_0$. A predictor-corrector method was used here.

REFERENCE

Ackerberg, R. C. 1971 *J. Fluid Mech.* **46**, 727.